GUIDE 98-3

Uncertainty of measurement —

Part 3:
Guide to the expression of uncertainty in measurement (GUM:1995)

Incertitude de mesure —

Partie 3: Guide pour l'expression de l'incertitude de mesure (GUM:1995)

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This *Guide* establishes general rules for evaluating and expressing uncertainty in measurement that are intended to be applicable to a broad spectrum of measurements. The basis of the *Guide* is Recommendation 1 (CI-1981) of the Comité International des Poids et Mesures (CIPM) and Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties. The Working Group was convened by the Bureau International des Poids et Mesures (BIPM) in response to a request of the CIPM. The CIPM Recommendation is the only recommendation concerning the expression of uncertainty in measurement adopted by an intergovernmental organization.

This *Guide* was prepared by a joint working group consisting of experts nominated by the BIPM, the International Electrotechnical Commission (IEC), the International Organization for Standardization (ISO), and the International Organization of Legal Metrology (OIML).

The following seven organizations* supported the development of this *Guide*, which is published in their name:

BIPM: Bureau International des Poids et Mesures

IEC: International Electrotechnical Commission

IFCC: International Federation of Clinical Chemistry**

ISO: International Organization for Standardization

IUPAC: International Union of Pure and Applied Chemistry**

IUPAP: International Union of Pure and Applied Physics**

OIML: International Organization of Legal Metrology

Users of this *Guide* are invited to send their comments and requests for clarification to any of the seven supporting organizations, the mailing addresses of which are given on the inside front cover***.

* **Footnote to the 2008 version:**

In 2005, the International Laboratory Accreditation Cooperation (ILAC) officially joined the seven founding international organizations.

** **Footnote to the 2008 version:**

The names of these three organizations have changed since 1995. They are now:
IFCC: International Federation for Clinical Chemistry and Laboratory Medicine
IUPAC: International Organization for Pure and Applied Chemistry
IUPAP: International Organization for Pure and Applied Physics.

*** **Footnote to the 2008 version:**

Links to the addresses of the eight organizations presently involved in the JCGM (Joint Committee for Guides in Metrology) are given on http://www.bipm.org/en/committees/jc/jcgm.

Foreword

In 1977, recognizing the lack of international consensus on the expression of uncertainty in measurement, the world's highest authority in metrology, the Comité International des Poids et Mesures (CIPM), requested the Bureau International des Poids et Mesures (BIPM) to address the problem in conjunction with the national standards laboratories and to make a recommendation.

The BIPM prepared a detailed questionnaire covering the issues involved and distributed it to 32 national metrology laboratories known to have an interest in the subject (and, for information, to five international organizations). By early 1979 responses were received from 21 laboratories [1].[1)] Almost all believed that it was important to arrive at an internationally accepted procedure for expressing measurement uncertainty and for combining individual uncertainty components into a single total uncertainty. However, a consensus was not apparent on the method to be used. The BIPM then convened a meeting for the purpose of arriving at a uniform and generally acceptable procedure for the specification of uncertainty; it was attended by experts from 11 national standards laboratories. This Working Group on the Statement of Uncertainties developed Recommendation INC-1 (1980), Expression of Experimental Uncertainties [2]. The CIPM approved the Recommendation in 1981 [3] and reaffirmed it in 1986 [4].

The task of developing a detailed guide based on the Working Group Recommendation (which is a brief outline rather than a detailed prescription) was referred by the CIPM to the International Organization for Standardization (ISO), since ISO could better reflect the needs arising from the broad interests of industry and commerce.

Responsibility was assigned to the ISO Technical Advisory Group on Metrology (TAG 4) because one of its tasks is to coordinate the development of guidelines on measurement topics that are of common interest to ISO and the six organizations that participate with ISO in the work of TAG 4: the International Electrotechnical Commission (IEC), the partner of ISO in worldwide standardization; the CIPM and the International Organization of Legal Metrology (OIML), the two worldwide metrology organizations; the International Union of Pure and Applied Chemistry (IUPAC) and the International Union of Pure and Applied Physics (IUPAP), the two international unions that represent chemistry and physics; and the International Federation of Clinical Chemistry (IFCC).

TAG 4 in turn established Working Group 3 (ISO/TAG 4/WG 3) composed of experts nominated by the BIPM, IEC, ISO, and OIML and appointed by the Chairman of TAG 4. It was assigned the following terms of reference:

> To develop a guidance document based upon the recommendation of the BIPM Working Group on the Statement of Uncertainties which provides rules on the expression of measurement uncertainty for use within standardization, calibration, laboratory accreditation, and metrology services;
>
> The purpose of such guidance is

- to promote full information on how uncertainty statements are arrived at;
- to provide a basis for the international comparison of measurement results.

This first edition of ISO/IEC Guide 98-3 cancels and replaces the *Guide to the Expression of Uncertainty in Measurement (GUM)*, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1993, corrected and reprinted in 1995.

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1) See the Bibliography.

* **Footnote to the 2008 version:**

In producing this 2008 version of the GUM, necessary corrections only to the printed 1995 version have been introduced by JCGM/WG 1. These corrections occur in Subclauses 4.2.2, 4.2.4, 5.1.2, B.2.17, C.3.2, C.3.4, E.4.3, H.4.3, H.5.2.5 and H.6.2.

0 Introduction

0.1 When reporting the result of a measurement of a physical quantity, it is obligatory that some quantitative indication of the quality of the result be given so that those who use it can assess its reliability. Without such an indication, measurement results cannot be compared, either among themselves or with reference values given in a specification or standard. It is therefore necessary that there be a readily implemented, easily understood, and generally accepted procedure for characterizing the quality of a result of a measurement, that is, for evaluating and expressing its *uncertainty*.

0.2 The concept of *uncertainty* as a quantifiable attribute is relatively new in the history of measurement, although *error* and *error analysis* have long been a part of the practice of measurement science or metrology. It is now widely recognized that, when all of the known or suspected components of error have been evaluated and the appropriate corrections have been applied, there still remains an uncertainty about the correctness of the stated result, that is, a doubt about how well the result of the measurement represents the value of the quantity being measured.

0.3 Just as the nearly universal use of the International System of Units (SI) has brought coherence to all scientific and technological measurements, a worldwide consensus on the evaluation and expression of uncertainty in measurement would permit the significance of a vast spectrum of measurement results in science, engineering, commerce, industry, and regulation to be readily understood and properly interpreted. In this era of the global marketplace, it is imperative that the method for evaluating and expressing uncertainty be uniform throughout the world so that measurements performed in different countries can be easily compared.

0.4 The ideal method for evaluating and expressing the uncertainty of the result of a measurement should be:

— *universal*: the method should be applicable to all kinds of measurements and to all types of input data used in measurements.

The actual quantity used to express uncertainty should be:

— *internally consistent*: it should be directly derivable from the components that contribute to it, as well as independent of how these components are grouped and of the decomposition of the components into subcomponents;

— *transferable*: it should be possible to use directly the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement in which the first result is used.

Further, in many industrial and commercial applications, as well as in the areas of health and safety, it is often necessary to provide an interval about the measurement result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the quantity subject to measurement. Thus the ideal method for evaluating and expressing uncertainty in measurement should be capable of readily providing such an interval, in particular, one with a coverage probability or level of confidence that corresponds in a realistic way with that required.

0.5 The approach upon which this guidance document is based is that outlined in Recommendation INC-1 (1980) [2] of the Working Group on the Statement of Uncertainties, which was convened by the BIPM in response to a request of the CIPM (see Foreword). This approach, the justification of which is discussed in Annex E, meets all of the requirements outlined above. This is not the case for most other methods in current use. Recommendation INC-1 (1980) was approved and reaffirmed by the CIPM in its own Recommendations 1 (CI-1981) [3] and 1 (CI-1986) [4]; the English translations of these CIPM Recommendations are reproduced in Annex A (see A.2 and A.3, respectively). Because Recommendation INC-1 (1980) is the foundation upon which this document rests, the English translation is reproduced in 0.7 and the French text, which is authoritative, is reproduced in A.1.

0.6 A succinct summary of the procedure specified in this guidance document for evaluating and expressing uncertainty in measurement is given in Clause 8 and a number of examples are presented in detail in Annex H. Other annexes deal with general terms in metrology (Annex B); basic statistical terms and concepts (Annex C); "true" value, error, and uncertainty (Annex D); practical suggestions for evaluating uncertainty components (Annex F); degrees of freedom and levels of confidence (Annex G); the principal mathematical symbols used throughout the document (Annex J); and bibliographical references (Bibliography). An alphabetical index concludes the document.

0.7 Recommendation INC-1 (1980) Expression of experimental uncertainties

1) The uncertainty in the result of a measurement generally consists of several components which may be grouped into two categories according to the way in which their numerical value is estimated:

   A. those which are evaluated by statistical methods,

   B. those which are evaluated by other means.

   There is not always a simple correspondence between the classification into categories A or B and the previously used classification into "random" and "systematic" uncertainties. The term "systematic uncertainty" can be misleading and should be avoided.

   Any detailed report of the uncertainty should consist of a complete list of the components, specifying for each the method used to obtain its numerical value.

2) The components in category A are characterized by the estimated variances s_i^2, (or the estimated "standard deviations" s_i) and the number of degrees of freedom v_i. Where appropriate, the covariances should be given.

3) The components in category B should be characterized by quantities u_j^2, which may be considered as approximations to the corresponding variances, the existence of which is assumed. The quantities u_j^2 may be treated like variances and the quantities u_j like standard deviations. Where appropriate, the covariances should be treated in a similar way.

4) The combined uncertainty should be characterized by the numerical value obtained by applying the usual method for the combination of variances. The combined uncertainty and its components should be expressed in the form of "standard deviations".

5) If, for particular applications, it is necessary to multiply the combined uncertainty by a factor to obtain an overall uncertainty, the multiplying factor used must always be stated.

Uncertainty of measurement —

Part 3:
Guide to the expression of uncertainty in measurement (GUM:1995)

1 Scope

1.1 This *Guide* establishes general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields — from the shop floor to fundamental research. Therefore, the principles of this *Guide* are intended to be applicable to a broad spectrum of measurements, including those required for:

— maintaining quality control and quality assurance in production;

— complying with and enforcing laws and regulations;

— conducting basic research, and applied research and development, in science and engineering;

— calibrating standards and instruments and performing tests throughout a national measurement system in order to achieve traceability to national standards;

— developing, maintaining, and comparing international and national physical reference standards, including reference materials.

1.2 This *Guide* is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity — the measurand — that can be characterized by an essentially unique value. If the phenomenon of interest can be represented only as a distribution of values or is dependent on one or more parameters, such as time, then the measurands required for its description are the set of quantities describing that distribution or that dependence.

1.3 This *Guide* is also applicable to evaluating and expressing the uncertainty associated with the conceptual design and theoretical analysis of experiments, methods of measurement, and complex components and systems. Because a measurement result and its uncertainty may be conceptual and based entirely on hypothetical data, the term "result of a measurement" as used in this *Guide* should be interpreted in this broader context.

1.4 This *Guide* provides general rules for evaluating and expressing uncertainty in measurement rather than detailed, technology-specific instructions. Further, it does not discuss how the uncertainty of a particular measurement result, once evaluated, may be used for different purposes, for example, to draw conclusions about the compatibility of that result with other similar results, to establish tolerance limits in a manufacturing process, or to decide if a certain course of action may be safely undertaken. It may therefore be necessary to develop particular standards based on this *Guide* that deal with the problems peculiar to specific fields of measurement or with the various uses of quantitative expressions of uncertainty.* These standards may be simplified versions of this *Guide* but should include the detail that is appropriate to the level of accuracy and complexity of the measurements and uses addressed.

NOTE There may be situations in which the concept of uncertainty of measurement is believed not to be fully applicable, such as when the precision of a test method is determined (see Reference [5], for example).

* **Footnote to the 2008 version:**

Several derivative general and specific applications documents have been published. Non-exhaustive compilations listing these documents can be found on http://www.bipm.org/en/committees/jc/jcgm/wg1_bibliography.html. In addition, up-to-date listings of documents that cite the *Guide to the expression of uncertainty in measurement* can be found by using the full-text search options on http://www.iso.org/ and http://www.iec.ch/.

