



# IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

---

**IEEE Power & Energy Society**

Sponsored by the  
Power System Instrumentation and Measurements Committee

1459<sup>TM</sup>

IEEE  
3 Park Avenue  
New York, NY 10016-5997, USA

19 March 2010

**IEEE Std 1459<sup>TM</sup>-2010**  
(Revision of  
IEEE Std 1459-2000)



# **IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions**

Sponsor

**Power System Instrumentation and Measurements Committee**

of the

**IEEE Power & Energy Society**

Approved 2 February 2010

**IEEE-SA Standards Board**

Figure 1 © 1983 IEEE. Reprinted, with permission, from the IEEE and R. H. Stevens.

**Abstract:** Definitions used for measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions are provided in this standard. Mathematical expressions that were used in the past, as well as new expressions, are listed, as well as explanations of the features of the new definitions.

**Keywords:** active power, apparent power, nonactive power, power factor, reactive power, total harmonic distortion

---

The Institute of Electrical and Electronics Engineers, Inc.  
3 Park Avenue, New York, NY 10016-5997, USA

Copyright © 2010 by the Institute of Electrical and Electronics Engineers, Inc.  
All rights reserved. Published 19 March 2010. Printed in the United States of America.

IEEE is a registered trademark in the U.S. Patent & Trademark Office, owned by the Institute of Electrical and Electronics Engineers, Incorporated.

**PDF:** ISBN 978-0-7381-6058-0      STD95967  
**Print:** ISBN 978-0-7381-6059-7      STDPD95967

*IEEE prohibits discrimination, harassment and bullying. For more information, visit <http://www.ieee.org/web/aboutus/whatis/policies/p9-26.html>. No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.*

**IEEE Standards** documents are developed within the IEEE Societies and the Standards Coordinating Committees of the IEEE Standards Association (IEEE-SA) Standards Board. The IEEE develops its standards through a consensus development process, approved by the American National Standards Institute, which brings together volunteers representing varied viewpoints and interests to achieve the final product. Volunteers are not necessarily members of the Institute and serve without compensation. While the IEEE administers the process and establishes rules to promote fairness in the consensus development process, the IEEE does not independently evaluate, test, or verify the accuracy of any of the information or the soundness of any judgments contained in its standards.

Use of an IEEE Standard is wholly voluntary. The IEEE disclaims liability for any personal injury, property or other damage, of any nature whatsoever, whether special, indirect, consequential, or compensatory, directly or indirectly resulting from the publication, use of, or reliance upon this, or any other IEEE Standard document.

The IEEE does not warrant or represent the accuracy or content of the material contained herein, and expressly disclaims any express or implied warranty, including any implied warranty of merchantability or fitness for a specific purpose, or that the use of the material contained herein is free from patent infringement. IEEE Standards documents are supplied “**AS IS.**”

The existence of an IEEE Standard does not imply that there are no other ways to produce, test, measure, purchase, market, or provide other goods and services related to the scope of the IEEE Standard. Furthermore, the viewpoint expressed at the time a standard is approved and issued is subject to change brought about through developments in the state of the art and comments received from users of the standard. Every IEEE Standard is subjected to review at least every five years for revision or reaffirmation, or every ten years for stabilization. When a document is more than five years old and has not been reaffirmed, or more than ten years old and has not been stabilized, it is reasonable to conclude that its contents, although still of some value, do not wholly reflect the present state of the art. Users are cautioned to check to determine that they have the latest edition of any IEEE Standard.

In publishing and making this document available, the IEEE is not suggesting or rendering professional or other services for, or on behalf of, any person or entity. Nor is the IEEE undertaking to perform any duty owed by any other person or entity to another. Any person utilizing this, and any other IEEE Standards document, should rely upon his or her independent judgment in the exercise of reasonable care in any given circumstances or, as appropriate, seek the advice of a competent professional in determining the appropriateness of a given IEEE standard.

**Interpretations:** Occasionally questions may arise regarding the meaning of portions of standards as they relate to specific applications. When the need for interpretations is brought to the attention of IEEE, the Institute will initiate action to prepare appropriate responses. Since IEEE Standards represent a consensus of concerned interests, it is important to ensure that any interpretation has also received the concurrence of a balance of interests. For this reason, IEEE and the members of its societies and Standards Coordinating Committees are not able to provide an instant response to interpretation requests except in those cases where the matter has previously received formal consideration. A statement, written or oral, that is not processed in accordance with the IEEE-SA Standards Board Operations Manual shall not be considered the official position of IEEE or any of its committees and shall not be considered to be, nor be relied upon as, a formal interpretation of the IEEE. At lectures, symposia, seminars, or educational courses, an individual presenting information on IEEE standards shall make it clear that his or her views should be considered the personal views of that individual rather than the formal position, explanation, or interpretation of the IEEE.

Comments for revision of IEEE Standards are welcome from any interested party, regardless of membership affiliation with IEEE. Suggestions for changes in documents should be in the form of a proposed change of text, together with appropriate supporting comments. Recommendations to change the status of a stabilized standard should include a rationale as to why a revision or withdrawal is required. Comments and recommendations on standards, and requests for interpretations should be addressed to:

Secretary, IEEE-SA Standards Board  
445 Hoes Lane  
Piscataway, NJ 08854  
USA

Authorization to photocopy portions of any individual standard for internal or personal use is granted by The Institute of Electrical and Electronics Engineers, Inc., provided that the appropriate fee is paid to Copyright Clearance Center. To arrange for payment of licensing fee, please contact Copyright Clearance Center, Customer Service, 222 Rosewood Drive, Danvers, MA 01923 USA; +1 978 750 8400. Permission to photocopy portions of any individual standard for educational classroom use can also be obtained through the Copyright Clearance Center.

## Introduction

This introduction is not part of IEEE Std 1459-2010, IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions.

The definitions for active, reactive, and apparent powers that are currently used are based on the knowledge developed and agreed on during the 1940s. Such definitions served the industry well, as long as the current and voltage waveforms remained nearly sinusoidal.

Important changes have occurred in the last 50 years. The new environment is conditioned by the following facts:

- a) Power electronics equipment, such as Adjustable Speed Drives, Controlled Rectifiers, Cycloconverters, Electronically Ballasted Lamps, Arc and Induction Furnaces, and clusters of Personal Computers, represent major nonlinear and parametric loads proliferating among industrial and commercial customers. Such loads have the potential to create a host of disturbances for the utility and the end-user's equipment. The main problems stem from the flow of nonactive energy caused by harmonic currents and voltages.
- b) New definitions of powers have been discussed in the last 30 years in the engineering literature (Filipski and Labaj [B9]<sup>a</sup>). The mechanism of electric energy flow for nonsinusoidal and/or unbalanced conditions is well understood today.
- c) The traditional instrumentation designed for the sinusoidal 60/50 Hz waveform is prone to significant errors when the current and the voltage waveforms are distorted (Filipski and Labaj [B9]).
- d) Microprocessors and minicomputers enable today's manufacturers of electrical instruments to construct new, accurate, and versatile metering equipment that is capable of measuring electrical quantities defined by means of advanced mathematical models.
- e) There is a need to quantify correctly the distortions caused by the nonlinear and parametric loads, and to apply a fair distribution of the financial burden required to maintain the quality of electric service.

This standard lists new definitions of powers needed for the following particular situations:

- When the voltage and current waveforms are nonsinusoidal
- When the load is unbalanced or the supplying voltages are asymmetrical
- When the energy dissipated in the neutral path due to zero-sequence current components has economical significance

The new definitions were developed to give guidance with respect to the quantities that should be measured or monitored for revenue purposes, engineering economic decisions, and determination of major harmonic polluters. The following important electrical quantities are recognized by this standard:

- The power frequency (60/50 Hz or fundamental) of apparent, active, and reactive powers. These three basic quantities are the quintessence of the power flow in electric networks. They define what is generated, transmitted, distributed, and sold by the electric utilities and bought by the end users. This is the electric energy transmitted by the 60/50 Hz electromagnetic field. In poly-phase systems, the power frequency positive-sequence powers are the important dominant quantities. The power frequency positive-sequence power factor is a key value that helps determine and adjust the flow of power frequency positive-sequence reactive power. The

---

<sup>a</sup> The numbers in brackets correspond to those of the bibliography in Annex C.

fundamental positive-sequence reactive power is of utmost importance in power systems; it governs the fundamental voltage magnitude and its distribution along the feeders and affects electromechanical stability as well as the energy loss.

- The effective apparent power in three-phase systems is  $S_e = 3V_e I_e$ , where  $V_e$  and  $I_e$  are the equivalent voltage and current. In sinusoidal and balanced situations,  $S_e$  is equal to the conventional apparent power  $S = 3V_{\ell n} I = \sqrt{3}V_{\ell \ell} I$ , where  $V_{\ell n}$  and  $V_{\ell \ell}$  are the line-to-neutral and the line-to-line voltage, respectively. For sinusoidal unbalanced or for nonsinusoidal balanced or unbalanced situations,  $S_e$  allows rational and correct computation of the power factor. This quantity was proposed in 1922 by the German engineer Buchholz [B1] and in 1933 was explained by the American engineer Goodhue [B11].
- The non-60 Hz or nonfundamental apparent power is  $S_N$  (for brevity, 50 Hz power is not always mentioned). This power quantifies the overall amount of harmonic pollution delivered or absorbed by a load. It also quantifies the required capacity of dynamic compensators or active filters when used for nonfundamental compensation alone.
- Current distortion power  $D_I$  identifies the segment of nonfundamental nonactive power due to current distortion. This is usually the dominant component of  $S_N$ .
- Voltage distortion power  $D_V$  separates the nonfundamental nonactive power component due to voltage distortion.
- Apparent harmonic power  $S_H$  indicates the level of apparent power due to harmonic voltages and currents alone. This is the smallest component of  $S_N$  and includes the harmonic active power  $P_H$ .

To avoid confusion, it was decided not to add new units. The use of the watts (W) for instantaneous and active powers, volt-amperes (VA) for apparent powers, and (var) for all the nonactive powers maintains the distinct separation among these three major types of powers.

There is not yet available a generalized power theory that can provide a simultaneous common base for

- Energy billing
- Evaluation of electric energy quality
- Detection of the major sources of waveform distortion
- Theoretical calculations for the design of mitigation equipment such as active filters or dynamic compensators

This standard is meant to provide definitions extended from the well-established concepts. It is meant to serve the user who wants to measure and design instrumentation for energy and power quantification. It is not meant to help in the design of real-time control of dynamic compensators or for diagnosis instrumentation used to pinpoint to a specific type of annoying event or harmonic.

These definitions are meant to serve as a guideline and as a useful benchmark for future developments.

## Notice to users

## Laws and regulations

Users of these documents should consult all applicable laws and regulations. Compliance with the provisions of this standard does not imply compliance to any applicable regulatory requirements. Implementers of the standard are responsible for observing or referring to the applicable regulatory

requirements. IEEE does not, by the publication of its standards, intend to urge action that is not in compliance with applicable laws, and these documents may not be construed as doing so.

## Copyrights

This document is copyrighted by the IEEE. It is made available for a wide variety of both public and private uses. These include both use, by reference, in laws and regulations, and use in private self-regulation, standardization, and the promotion of engineering practices and methods. By making this document available for use and adoption by public authorities and private users, the IEEE does not waive any rights in copyright to this document.

## Updating of IEEE documents

Users of IEEE standards should be aware that these documents may be superseded at any time by the issuance of new editions or may be amended from time to time through the issuance of amendments, corrigenda, or errata. An official IEEE document at any point in time consists of the current edition of the document together with any amendments, corrigenda, or errata then in effect. In order to determine whether a given document is the current edition and whether it has been amended through the issuance of amendments, corrigenda, or errata, visit the IEEE Standards Association web site at <http://ieeexplore.ieee.org/xpl/standards.jsp>, or contact the IEEE at the address listed previously.

For more information about the IEEE Standards Association or the IEEE standards development process, visit the IEEE-SA web site at <http://standards.ieee.org>.

## Errata

Errata, if any, for this and all other standards can be accessed at the following URL: <http://standards.ieee.org/reading/ieee/updates/errata/index.html>. Users are encouraged to check this URL for errata periodically.

## Interpretations

Current interpretations can be accessed at the following URL: <http://standards.ieee.org/reading/ieee/interp/index.html>.

## Patents

Attention is called to the possibility that implementation of this standard may require use of subject matter covered by patent rights. By publication of this standard, no position is taken with respect to the existence or validity of any patent rights in connection therewith. The IEEE is not responsible for identifying Essential Patent Claims for which a license may be required, for conducting inquiries into the legal validity or scope of Patents Claims or determining whether any licensing terms or conditions provided in connection with submission of a Letter of Assurance, if any, or in any licensing agreements are reasonable or non-discriminatory. Users of this standard are expressly advised that determination of the validity of any patent rights, and the risk of infringement of such rights, is entirely their own responsibility. Further information may be obtained from the IEEE Standards Association.



## Participants

At the time this standard was submitted to the IEEE-SA Standards Board for approval, the Non-Sinusoidal Situations Working Group had the following membership:

**Alexander E. Emanuel**, *Chair*

**Eddy So**, *Sponsor*

Jose Policarpo Abreu  
Rejean Arseneau  
Santiago Barcon  
Andrew Berrisford  
Yahia Baghzouz  
Keneth B. Bowes  
James A. Braun  
Antonio Cataliotti  
David Cooper  
Valentina Cosentino  
Mikey D. Cox  
Roger H. Daugherty  
Soni Devendra  
Dario Di Cara  
William Dickerson  
Alexander Domijan

David Elmore  
Gaetan Ethier  
Erich Gunther  
Dennis Hansen  
Ernst Hanique  
Gilbert C. Hensley  
John Houdek  
Roberto Langella  
Michael Lowenstein  
William Moncrief  
Alexander McEachern  
Dalgerti Milanez  
Thomas L. Nelson  
Vuong Nguyen  
Daniel Nordell  
Salvatore Nuccio

Slobodan Pajic  
Lorenzo Peretto  
Jan H. C. Pretorius  
Paulo Ribeiro  
Daniel Sabin  
Kalyan Sen  
Piet H. Swart  
Donald Tandon  
Alfredo Testa  
Grazia Todeschini  
Daniel Ward  
Scott Weikel  
Stephan Weiss  
Douglas Williams  
Jacques L. Willems  
Daan van Wyk

The following members of the individual balloting committee voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

William J. Ackerman  
Ali Al Awazi  
David Baron  
Steven Brockschink  
William Brumsickle  
Gustavo Brunello  
Yunxiang Chen  
John Cooper  
Tommy Cooper  
John Crouse  
Roger H. Daugherty  
Gary L. Donner  
Neal Dowling  
Dana Dufield  
Gearold O. H. Eidhin  
Alexander E. Emanuel  
Gary Engmann  
Paul Forquer  
Marcel Fortin  
Randall Groves  
Gary Heuston

Werner Hoelzl  
Randy Horton  
Innocent Kamwa  
Piotr Karocki  
Jon Kay  
Tanuj Khandelwal  
Yuri Khersonsky  
Harold Kirkham  
Joseph L. Koepfinger  
Jim Kulchisky  
Federico Lopez  
Michael Lowenstein  
Keith Malmedal  
Jose Marrero  
Kenneth Martin  
William McBride  
Kenneth Mcclenahan  
Gary Michel  
Charles Morse  
Jerry Murphy  
Bruce Muschlit

Michael S. Newman  
David Nichols  
Ulrich Pohl  
Iulian Profir  
Michael Roberts  
Charles Rogers  
Bob Saint  
Steven Sano  
Bartien Sayogo  
Thomas Schossig  
Kenneth Sedziol  
Ahmed El Serafi  
James E. Smith  
Aaron Snyder  
Eddy So  
Michael Swearingen  
David Tepen  
John Vergis  
Scott Weikel  
James Wilson  
Ahmed Zobaa

When the IEEE-SA Standards Board approved this standard on 2 February 2010, it had the following membership:

**Robert M. Grow**, *Chair*  
**Tom A. Prevost**, *Vice Chair*  
**Steve M. Mills**, *Past Chair*  
**Judith Gorman**, *Secretary*

John Barr  
Karen Bartelson  
Victor Berman  
Ted Burse  
Richard DeBlasio  
Andrew Drozd  
Mark Epstein

Alexander Gelman  
James Hughes  
Richard H. Hulett  
Young Kyun Kim  
Joseph L. Koepfinger\*  
John Kulick

David J. Law  
Ted Olsen  
Glenn Parsons  
Ronald C. Petersen  
Narayanan Ramachandran  
Jon Walter Rosdahl  
Sam Sciacca

\*Member Emeritus

Also included are the following nonvoting IEEE-SA Standards Board liaisons:

Howard L. Wolfman, *TAB Representative*  
Michael Janezic, *NIST Representative*  
Satish K. Aggarwal, *NRC Representative*

Lorraine Patsco  
*IEEE Standards Program Manager, Document Development*

Matthew J. Ceglia  
*IEEE Standards Program Manager, Technical Program Development*

## Contents

1. Overview .....	1
1.1 Scope .....	1
1.2 Purpose .....	2
2. Normative references.....	2
3. Definitions .....	2
3.1 Single phase.....	2
3.2 Three-phase systems.....	13
Annex A (informative) Theoretical examples .....	30
Annex B (informative) Practical studies and measurements: A detailed explanation of apparent power components.....	34
Annex C (informative) Bibliography.....	39



# IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

*IMPORTANT NOTICE: This standard is not intended to ensure safety, security, health, or environmental protection in all circumstances. Implementers of the standard are responsible for determining appropriate safety, security, environmental, and health practices or regulatory requirements.*

*This IEEE document is made available for use subject to important notices and legal disclaimers. These notices and disclaimers appear in all publications containing this document and may be found under the heading “Important Notice” or “Important Notices and Disclaimers Concerning IEEE Documents.” They can also be obtained on request from IEEE or viewed at <http://standards.ieee.org/IPR/disclaimers.html>*

## 1. Overview

This standard is divided into three clauses. Clause 1 lists the scope of this document. Clause 2 lists references to other standards that are useful in applying this standard. Clause 3 provides the definitions, among which there are several new expressions.

The preferred mathematical expressions recommended for the instrumentation design are marked with a || sign. The additional expressions are meant to reinforce the theoretical approach and to facilitate a better understanding of the explained concepts.

### 1.1 Scope

This document provides definitions of electric power to quantify the flow of electrical energy in single-phase and three-phase circuits under sinusoidal, nonsinusoidal, balanced, and unbalanced conditions.