

BS EN 61710:2013



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# Power law model — Goodness-of-fit tests and estimation methods

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**Power law model -  
Goodness-of-fit tests and estimation methods  
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Essais d'adéquation et méthodes  
d'estimation des paramètres  
(CEI 61710:2013)

Potenzgesetz-Modell -  
Anpassungstests und Schätzverfahren  
(IEC 61710:2013)

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Comité Européen de Normalisation Electrotechnique  
Europäisches Komitee für Elektrotechnische Normung

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## Foreword

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The following dates are fixed:

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- latest date by which the national standards conflicting with the document have to be withdrawn (dow) 2016-06-26

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IEC 61703	NOTE	Harmonised as EN 61703.
IEC 61164:2004	NOTE	Harmonised as EN 61164:2004 (not modified).

## **Annex ZA** (normative)

### **Normative references to international publications with their corresponding European publications**

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE When an international publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

<u>Publication</u>	<u>Year</u>	<u>Title</u>	<u>EN/HD</u>	<u>Year</u>
IEC 60050-191	1990	International Electrotechnical Vocabulary (IEV) - Chapter 191: Dependability and quality of service	-	-

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## INTRODUCTION

This International Standard describes the power law model and gives step-by-step directions for its use. There are various models for describing the reliability of repairable items, the power law model being one of the most widely used. This standard provides procedures to estimate the parameters of the power law model and to test the goodness-of-fit of the power law model to data, to provide confidence intervals for the failure intensity and prediction intervals for the length of time to future failures. An input is required consisting of a data set of times at which relevant failures occurred, or were observed, for a repairable item or a set of copies of the same item, and the time at which observation of the item was terminated, if different from the time of final failure. All output results correspond to the item type under consideration.

Some of the procedures can require computer programs, but these are not unduly complex. This standard presents algorithms from which computer programs should be easy to construct.

## POWER LAW MODEL – GOODNESS-OF-FIT TESTS AND ESTIMATION METHODS

### 1 Scope

This International Standard specifies procedures to estimate the parameters of the power law model, to provide confidence intervals for the failure intensity, to provide prediction intervals for the times to future failures, and to test the goodness-of-fit of the power law model to data from repairable items. It is assumed that the time to failure data have been collected from an item, or some identical items operating under the same conditions (e.g. environment and load).

### 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

### 3 Terms and definitions

For the purposes of this document, the terms and definitions of IEC 60050-191 apply.

### 4 Symbols and abbreviations

The following symbols and abbreviations apply:

$\beta$	shape parameter of the power law model
$\hat{\beta}$	estimated shape parameter of the power law model
$\beta_{LB}, \beta_{UB}$	lower, upper confidence limits for $\beta$
$C^2$	Cramer-von-Mises goodness-of-fit test statistic
$C_{1-\gamma}^2(M)$	critical value for the Cramer-von-Mises goodness-of-fit test statistic at $\gamma$ level of significance
$\chi^2$	Chi-square goodness-of-fit test statistic
$\chi_{\gamma}^2(\nu)$	$\gamma$ th fractile of the $\chi^2$ distribution with $\nu$ degrees of freedom
$d$	number of intervals for groups of failures
$E[N(t)]$	expected accumulated number of failures up to time $t$
$E[t_j]$	expected accumulated time to $j$ th failure